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1.1 $x \to \infty: \; f(x) = \dfrac{-16e^x \to \infty}{(e^x+1)^2 \to \infty} \xrightarrow{L.H.} \dfrac{-16e^x}{2(e^x+1)\cdot e^x} = \dfrac{-16}{2(e^x+1)\to\infty} \to 0^{(-)}$

1.2 $f(-x) = \dfrac{-16e^{-x}}{(e^{-x}+1)^2} = \dfrac{-16e^{-x}\cdot e^{2x}}{(e^{-x}+1)^2\cdot e^{2x}} = \dfrac{-16e^{+x}}{[(e^{-x}+1)e^x]^2} = \dfrac{-16e^x}{(1+e^x)^2}$

Wegen $f(-x)=f(x)$: Achsensym. z. y-Achse

Erweitern ist einzige Mögl. d. Umform → "alten" Zähler herstellen

1.3 $f(x) = -16\dfrac{e^x}{(e^x+1)^2} \Rightarrow f'(x) = -16\dfrac{(e^x+1)^{\cancel{2}}\cdot e^x - e^x\cancel{(e^x+1)}\cdot 2e^x}{(e^x+1)^{\cancel{4}\,3}}$

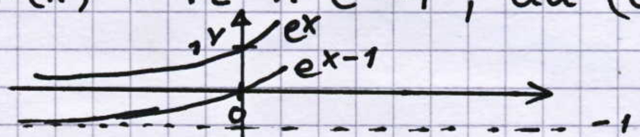
$e^{2x} = e^x\cdot e^x$

$= -16\dfrac{e^x(e^x+1) - 2e^{2x}}{(e^x+1)^3} = -16\dfrac{e^x(e^x+1-2e^x)}{(e^x+1)^3} = \dfrac{e^x(1-e^x)}{(e^x+1)^3}\cdot(-16)$

$f'(x) = 16\dfrac{(e^x-1)e^x}{(e^x+1)^3}$

$f'(x) = 0 \Rightarrow e^x - 1 = 0 \Leftrightarrow e^x = 1 \Leftrightarrow x = 0 \quad (e^0 = 1)$

VZ v. $f'(x)$ = VZ v. $e^x - 1$, da $(e^x+1)^3 > 0$



$f(0) = \dfrac{-16e^0}{(e^0+1)^2} = \dfrac{-16}{2^2}$

VZ $f'(x)$	−	0	+
G_f	smf	TIP	sms

⇒ TIP $(0|-4)$

G_f ist smf für $x \in]-\infty; 0]$

sms für $x \in [0; \infty[$

Intervalle verlangt ⇒ hinschreiben!

1.4

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1.5 $F'(x) \overset{!}{=} f(x) \Rightarrow F' = \dfrac{(e^x+b)\cdot 0 - a\cdot e^x}{(e^x+b)^2} = \dfrac{-ae^x}{(e^x+b)^2} \overset{!}{=} \dfrac{-16e^x}{(e^x+1)^2}$

Koeffizientenvergleich liefert $-a = -16 \Rightarrow \underline{a = 16}$; $\underline{b = 1}$

1.6 Gerade $g: y = -3$; Schnittstellen : $g(x) = f(x) \quad | \cdot (e^x+1)^2$

$-16e^x = -3(e^x+1)^2 \Leftrightarrow -16e^x = -3(e^x)^2 - 6e^x - 3$

$\Leftrightarrow 3(e^x)^2 - 10e^x + 3 = 0$; Subst : $e^x = u$

$\Rightarrow 3u^2 - 10u + 3 = 0 \Rightarrow u_{1/2} = \dfrac{5 \pm 4}{3}$ $u_1 = 3$; $u_2 = \frac{1}{3}$

Resubst : $e^x = u \Leftrightarrow x = \ln(u) \Rightarrow x_1 = \ln(3)$; $x_2 = \ln\left(\frac{1}{3}\right) = -\ln(3)$

Sym!

$I = 2 \cdot \int_0^{\ln(3)} (-3 - f(x))\,dx = 2 \cdot \left[-3x - F(x)\right]_0^{\ln(3)} = \left[G(x)\right]_0^{\ln(3)}$

$G(\ln(3)) = -3\ln(3) - \dfrac{16}{e^{\ln(3)}+1} = -3\ln(3) - \dfrac{16}{3+1}$

$G(0) = -3\cdot 0 - \dfrac{16}{e^0+1} = -\dfrac{16}{2} = -8$

$2\cdot\left[G(\ln(3)) - G(0)\right] = 2\cdot\left[-3\ln(3) - 4 + 8\right] = \underline{-6\ln(3) + 8} \approx 1{,}41$ [FE]

1.7.1 • y-Achse : $h(0) = \ln\left(\frac{16}{1+1}\right) = \ln(8) \Rightarrow \underline{S_y(0 / \ln(8))}$

• x-Achse : $h(x) = 0 \Rightarrow \text{Arg} = 1$ Also : $\underline{S_x(\ln(15) / 0)}$

$\dfrac{16}{e^x+1} = 1 \Leftrightarrow e^x + 1 = 16 \Leftrightarrow e^x = 15 \Rightarrow x = \ln(15)$

• $\underline{x \to \infty}$: $\ln\left(\dfrac{16}{e^x+1 \to \infty}\right) \to \text{"}\ln\left(\dfrac{16}{\infty}\right)\text{"} \to \text{"}\ln(0)\text{"} \to \underline{-\infty}$

• $\underline{x \to -\infty}$: $\ln\left(\dfrac{16}{e^x+1 \to 0+1}\right) \to \ln\left(\dfrac{16}{1}\right) = \underline{\ln(16)}$

Also : waagr. Tangente $y = \ln(16) \approx 2{,}77$

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1.7.2 $h'(x) = \frac{\cancel{(e^x+1)}}{\cancel{16}} \cdot \frac{0 - e^x \cdot \cancel{16}}{(e^x+1)^{\cancel{2}}} = \frac{-e^x}{e^x+1}$

$$h''(x) = \frac{(e^x+1)\cdot(-e^x) - (-e^x)(e^x)}{(e^x+1)^2} = \frac{-e^x\,(\overbrace{e^x+1-e^x}^{1})}{(e^x+1)^2}$$

$h''(x) < 0$ für alle $x \in \mathbb{R}$, weil $-e^x < 0$ und $(e^x+1)^2 > 0$

$\Rightarrow$ Rechtskrümmung in ganz $\mathbb{R}$; kein WEP

1.7.3

$\rightarrow G_h$

1.7.4 $P(0/y_P) = P(0/\ln(8))$ (vgl. 1.7.1)

$h'(0) = m_T = \frac{-e^0}{e^0+1} = \frac{-1}{1+1} = -\frac{1}{2} \Rightarrow m_N = 2 \quad (m_T \cdot m_N = -1)$

Normale $n(x) = 2x + \ln(8)$

NST d. Normalen: $n(x) = 0$, also $2x + \ln(8) = 0$

$\Rightarrow x_N = -\frac{1}{2}\ln(8)$

Flächenmaßzahl : $A = \frac{1}{2} g h = \frac{1}{2} \cdot \frac{1}{2}\ln(8) \cdot \ln(8)$

$A = \left(\frac{1}{2}\ln(8)\right)^2 \approx 1{,}08$